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DP-003-1164004

Seat No. ________

M. Sc. (Sem. IV) Examination

March - 2022

Mathematics : CMT-4004

(Graph Theory)

Faculty Code : 003
Subject Code : 1164004

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All the questions are compulsory.
(2) There are total five questions.
(3) Each question carries equal marks (14).

1 Answer any **seven** questions : **7×2=14**

(1) Define terms : Isomorphism of two graphs, Subgraph, Vertex disjoint subgraph and Edge disjoint subgraph.

(2) Define terms : Hamiltonian Cycle, Hamiltonian graph, Eulerian graph and Open Eulerian graph.

(3) Define term : Minimally connected graph. Also draw a graph G, with $|V(G)| \geq 5$ and G is a minimally connected graph.

(4) Give two non isomorphic graphs G_1 and G_2, which are having properties $|V(G_1)| = |V(G_2)|, |E(G_1)| = |E(G_2)|$ and for any non-negative integer t, the number of vertices in G_1 with degree t and the number of vertices in G_2 with degree t are same.

(5) State and prove, First Fundamental Theorem of Graph Theory.

(6) State Euler's Theorem.

(7) Write down terms : Fundamental cycle and Fundamental cut-set of a connected graph G with respect to a spanning tree T.

(8) Define term : Weighted graph and Minimal spanning tree.

(9) Define term: Edge connectivity and Vertex connectivity.

(10) Draw a graph G, so that the vertex connectivity for $G = 2$, the edge connectivity for $G = 3$ and $\delta(G) = \min_{v \in V(G)} d_G(v) = 4$.

2 Answer any **two** questions : **2×7=14**

(a) Let G be a finite graph. Prove that there are subgraphs $g_i = (V_i, E_i), i = 1, 2, ..., k$, for some $k \geq 1$ such that,

(i) Each g_i is a maximal connected subgraph of G.

(ii) $V_i \cap V_j = \phi, i \neq j$ and $i, j \in \{1, 2, ..., k\}$.

(iii) $V = V_1 \cup .... \cup V_k$ and $E = E_1 \cup .... \cup E_k$.

(iv) If $g = (W, F)$ be any connected sub graph of G, then g must be a subgraph g_i, for some $i \in \{1, 2, ..., k\}$.

(b) Let G be a simple graph with n vertices, q edges and k number of components in G. Prove that, $q \leq \frac{1}{2}(n-k)(n-k+1)$.

(c) Let G be graph and it does not contain any self loop. Suppose for any pair of vertices $u, v \in V(G)$, there is a unique path between u and v in G. Prove that, G is a tree.

3 Answer following **one** questions : **1×14=14**

(1) Let G be a connected graph with $E(G) \neq \phi$. Prove that G is an Eulerian graph if and only if it can be decomposed into edge disjoint cycles.

(2) Define term : Maximal non-Hamiltonian graph. Let G be a simple graph, $|V(G)| > 2$ and $d_G(v) \geq \frac{n}{2}, \forall\, v \in V(G)$. Prove that, G is a Hamiltonian graph.

4 Answer following **two** questions : **2×7=14**

(a) Let T be a tree and it has at least two vertices. Let $P = u_0 - u_1 - u_2 - .....u_n$ be a longest path in T. Prove that, u_0 and u_n both are pendent vertices in T.

(b) For a tree T, with $|V(T)| = n$, prove that T has $n-1$ edges.

(c) Prove that, a graph G is a minimally connected graph if and only if it is a tree.

5 Answer following **two** questions : **2×7=14**

(i) Let T be a tree with $|V(T)| \geq 2$. Prove that, T is a 2-chromatic graph.

(ii) Define adjacency matrix for a graph G. Write down adjacency matrix for C_6. Also write down at least four properties for the adjacency matrix $X(G)$, for a graph G.

(iii) Let G be a connected graph. Prove that, G is an Open Euler graph if and only if G has precisely two odd vertices and remaining are even vertices.

(iv) Let T be a tree with n vertices $(n \geq 2)$. Prove that, T has either one center or two centers. Also prove that, in the case of T has two centers, they must be adjacent by an edge in T.